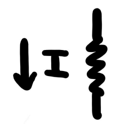
Week 2: Power and Equivalent Circuits

From last time:

- Ohm's Law:



$$\Delta V = IR$$

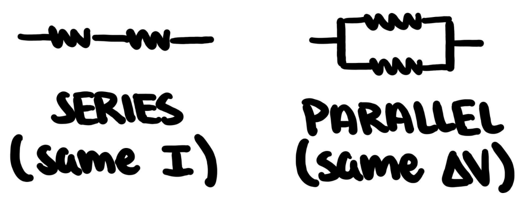
- KVL:

$$\sum_{\text{LOOP}} V = 0$$

- KCL:

$$\sum_{\text{IN}} I = \sum_{\text{OUT}} I$$

- Series and Parallel Resistances:

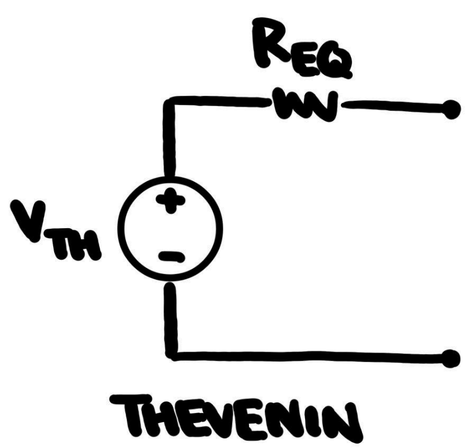


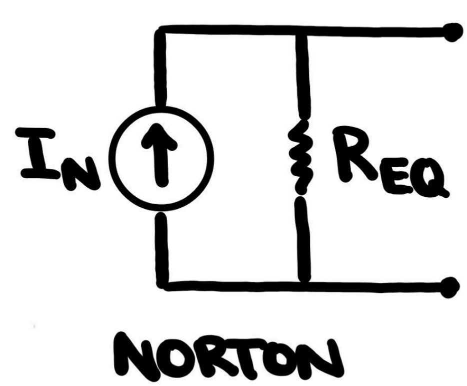
New for this week:

- Power (Watt's Law):

$$P_{\text{ABSORBED}} = \Delta V_{+ \to -} I$$

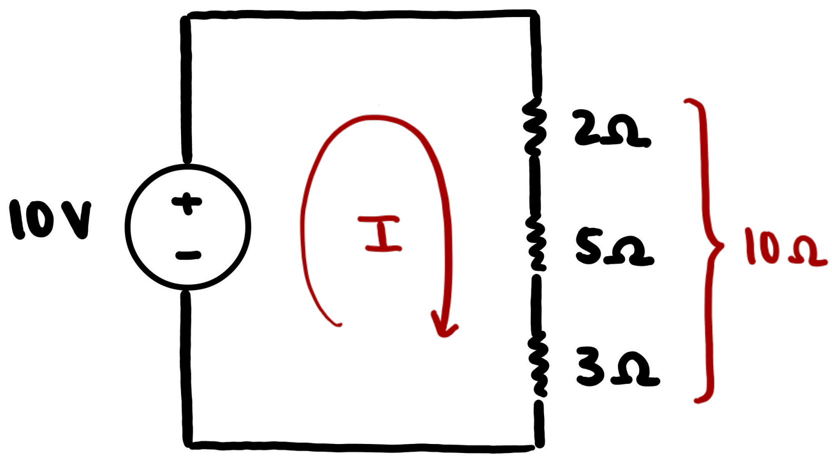
- Thevenin and Norton Equivalent:





Problems:

1. **Power Predicament:** Use KVL, Ohm's Law, and Watt's Law to find the power generated/consumed by each source and resistor in the below circuit:



$$R_{total} = 2\Omega + 5\Omega + 3\Omega = 10\Omega$$

$$\Delta V_{total} = IR_{total} \rightarrow I = \frac{\Delta V_{total}}{R_{total}} = \frac{10V}{10\Omega} = 1A$$

by voltage division, $\Delta V_{R_i} = \Delta V_{total}\left(\frac{R_i}{R_{total}}\right)$

↳ $\Delta V_{2\Omega} = 10V\left(\frac{2\Omega}{10\Omega}\right) = 2V$

↳ $\Delta V_{5\Omega} = 10V\left(\frac{5\Omega}{10\Omega}\right) = 5V$

↳ $\Delta V_{3\Omega} = 10V\left(\frac{3\Omega}{10\Omega}\right) = 3V$

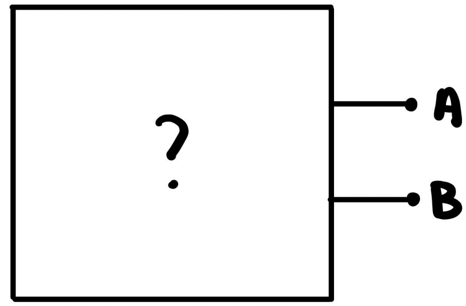
and so the dissipated/absorbed power for each resistor:

↳ $\boxed{P_{2\Omega} = 2V \cdot 1A = 2W}$

↳ $\boxed{P_{5\Omega} = 5V \cdot 1A = 5W}$

↳ $\boxed{P_{3\Omega} = 3V \cdot 1A = 3W}$

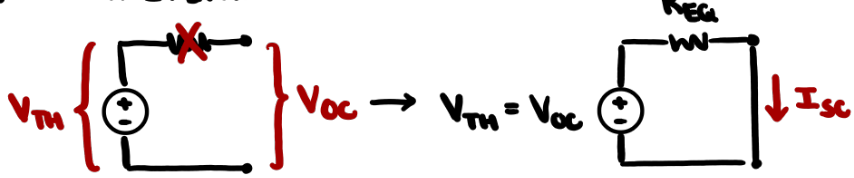
2. **Black Box Bonanza:** Consider the black box at right, which contains a circuit with two terminals, A and B, to connect to the outside world. We find the open circuit voltage and short circuit current between the terminals to be 7.5 V and 2.5 A, respectively.



a. Draw the Thevenin and Norton equivalent models of the internals of the black box. Compute the Thevenin voltage, Norton current, and equivalent resistance.

b. Demonstrate that the two circuit models are equivalent by calculating the power dissipated by a 2 Ω resistor connected between A and B.

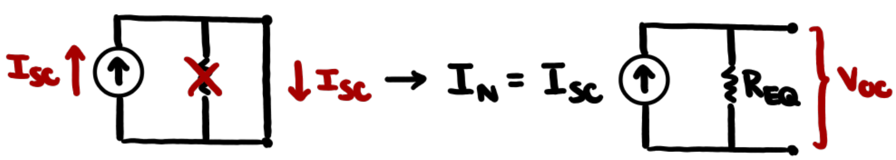
A] FOR THEVENIN:



$\Delta V = I_{SC} R_{EQ}$

$R_{EQ} = \frac{V_{OC}}{I_{SC}} = \frac{7.5V}{2.5A} = 3\Omega$

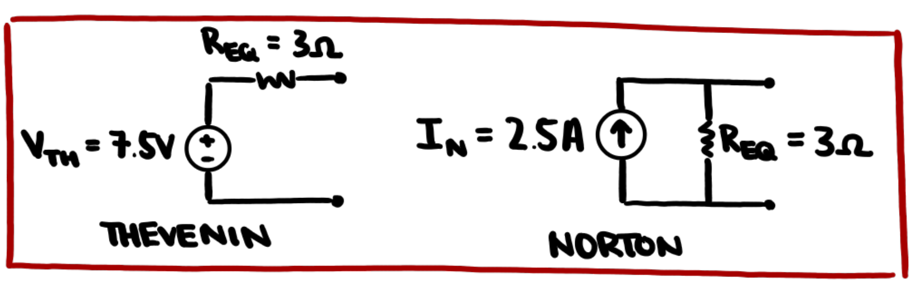
FOR NORTON:



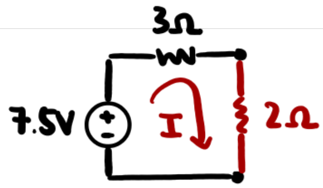
$\Delta V = I_N R_{EQ}$

$R_{EQ} = \frac{V_{OC}}{I_N} = \frac{7.5V}{2.5A} = 3\Omega$

THUS:



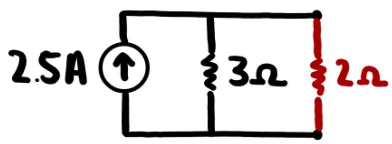
B] same P?



$$\Delta V = IR$$
$$I = \frac{\Delta V}{R} = \frac{7.5V}{5\Omega} = 1.5A$$

$$\Delta V_{2\Omega} = I \cdot 2\Omega$$
$$\Delta V_{2\Omega} = 1.5A \cdot 2\Omega$$
$$\Delta V_{2\Omega} = 3V$$

$$P_{THEVENIN} = \Delta V_{2\Omega} \cdot I = 3V \cdot 1.5A = 4.5W$$



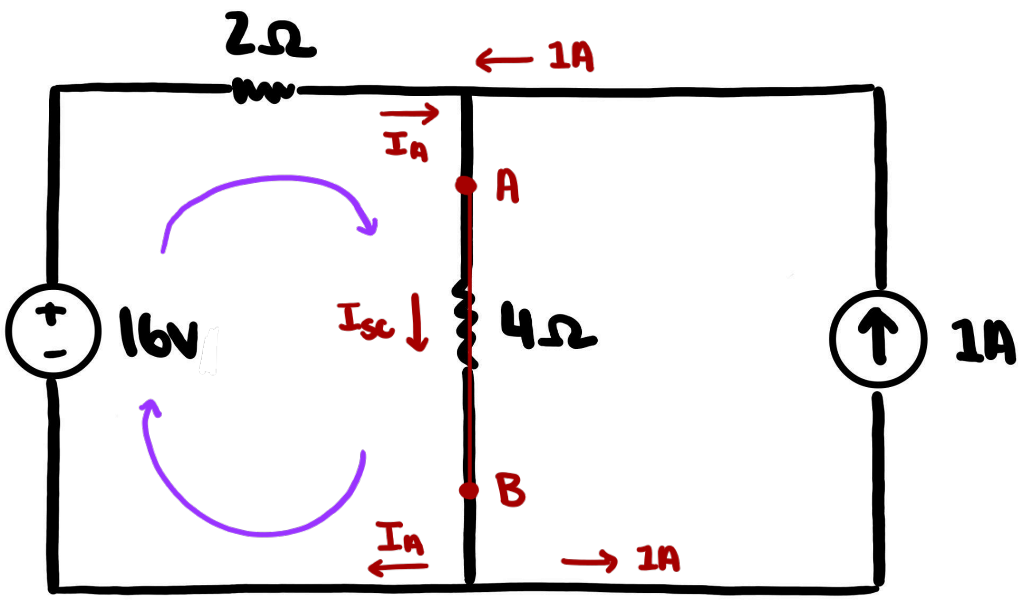
$$3\Omega \parallel 2\Omega = \frac{1}{\frac{1}{3\Omega} + \frac{1}{2\Omega}} = 1.2\Omega$$

$$\Delta V = IR = 2.5A \cdot 1.2\Omega = 3V$$
$$\hookrightarrow I = \frac{\Delta V_{2\Omega}}{2\Omega} = \frac{3V}{2\Omega} = 1.5A$$

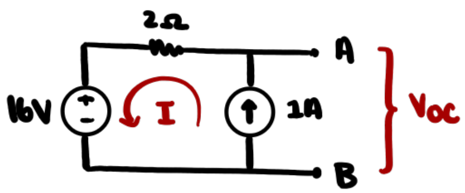
$$P_{NORTON} = \Delta V_{2\Omega} \cdot I = 3V \cdot 1.5A = 4.5W$$

SAME, YAY! :D

3. **Recitation Roadblock, Revisited**: Remember this tricky problem from last week? Use either a Thevenin or Norton equivalent circuit to simplify this circuit and solve for the voltage across and current through the center 4 Ω resistor.



- What is V_{oc}?



by KVL: $-16V - \cancel{I}^{\,1A} \cdot 2A + V_{oc} = 0$

$V_{oc} = 16V + 2V$

$V_{oc} = 18V$

- what is I_{sc}? see above KCL drawings...

$I_{sc} = I_A + 1A$

via purple KVL loop. $16V - I_A \cdot 2\Omega = 0A$

$I_A \cdot 2\Omega = 16V$

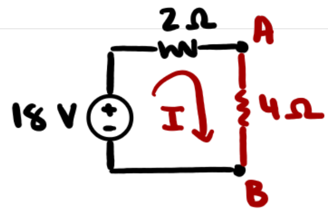
$I_A = \frac{16V}{2\Omega} = 8A$

$I_{sc} = 9A$

- Thevenin Equivalent:

$V_{TH} = V_{OC} = 18V$

$R_{EQ} = \frac{V_{OC}}{I_{SC}} = \frac{18V}{9A} = 2\Omega$



- then use KVL to find desired voltage/current:

$\Delta V = IR$

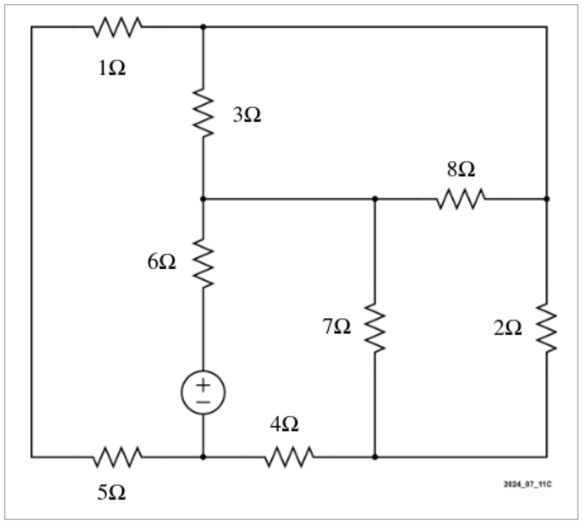
$I = \frac{\Delta V}{R} = \frac{18V}{6\Omega} = 3A$

$\Delta V = IR = 3A \cdot 4\Omega = 12V$

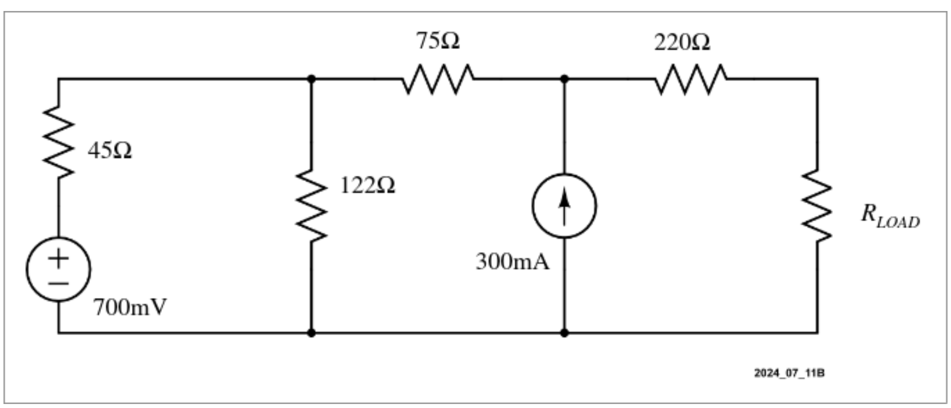
$$\boxed{\begin{array}{l} I_{4\Omega} = 3A \\ \Delta V_{4\Omega} = 12V \end{array}}$$

4. **(Bonus!) A Couple Challenge Problems:** The following are a few complex (taken from David's 18-220!) questions to probe your understanding of resistor networks, voltage/current division, and Norton/Thevenin equivalent circuits!

    a. Simplify the following network of resistors as much as you can: what values of resistors can be safely considered in series? Parallel?

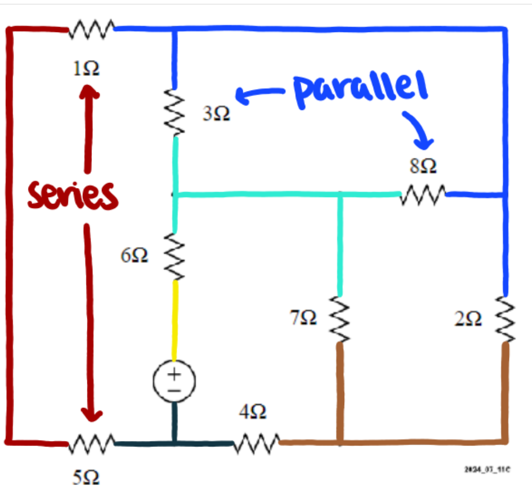


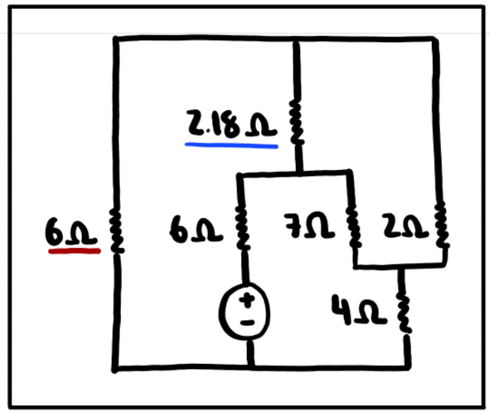
    b. Find the Thevenin equivalent of the following circuit with respect to the load resistor (*hint: it may be useful to look up a technique called "superposition" - you won't need it in this class, but it's an interesting alternative method of solving more complex circuits!)*:



    c. Call Kesden/David over and ask any questions you might have about ECE, life at CMU, job search, anything at all! :)

A]



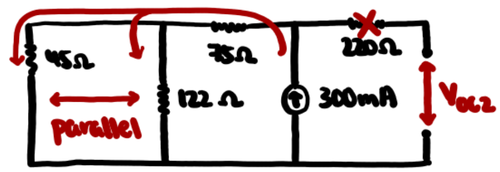


series: $5\Omega + 1\Omega = 6\Omega$

parallel: $3\Omega \parallel 8\Omega = \dfrac{1}{\frac{1}{3\Omega} + \frac{1}{8\Omega}} \approx 2.18\,\Omega$

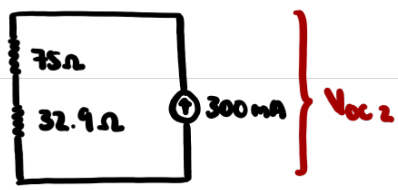
B]

want to find V_{oc}, I_{sc}; start with finding V_{oc} via superposition:



$V_{oc1} = V_s \cdot \dfrac{R_2}{R_1 + R_2}$

$V_{oc1} = 700 \cdot 10^{-3} \cdot \dfrac{122}{167} = 511\text{ mV}$



$\Delta V = IR$

$V_{oc2} = (300 \cdot 10^{-3})(107.9)$

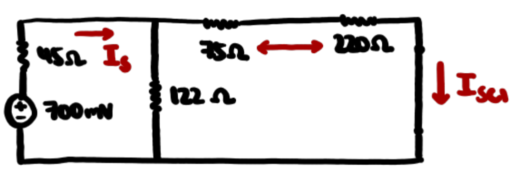
$V_{oc2} = 32.4\text{ V}$

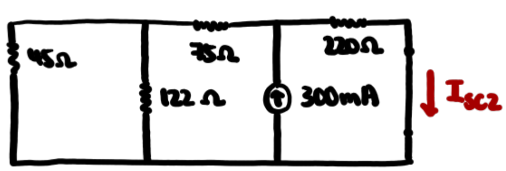
$V_{oc} = V_{oc1} + V_{oc2}$

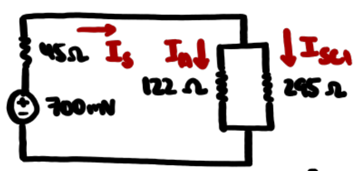
$V_{oc} = 511 \cdot 10^{-3} + 32.4$

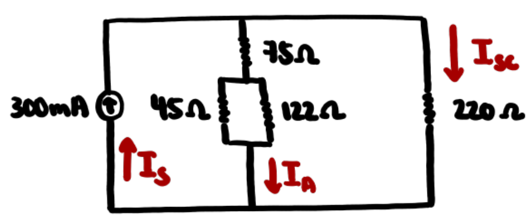
$\underline{V_{oc} = 32.9\text{ V}}$

now find I_{sc} by the same method:









$I_s = \dfrac{V_s}{R} = \dfrac{700 \cdot 10^{-3}}{45 + 86.3} = 5.33\text{ mA}$

$I_s = I_{sc} + I_A$

$V_A = V_{sc1} = I_s \cdot R_{eq1} = 5.33 \cdot 10^{-3} \cdot 86.3 = 460\text{ mV}$

$V_A = V_{sc}$; $I_s = I_A + I_{sc}$

$R_A = 75 + \dfrac{1}{\frac{1}{45} + \frac{1}{122}} = 108\,\Omega$

$\Delta V = IR$

$I_{sc1} = \frac{\Delta V_{sc1}}{R}$

$I_{sc1} = \frac{460 \cdot 10^{-3}}{295} = 1.56 \text{ mA}$

$I_s = I_A + I_{sc}$

$I_A(108) = I_{sc}(220)$

$I_A = I_s - I_{sc}$

$(I_s - I_{sc2}) \cdot 108 = I_{sc}(220)$

$I_{sc2} = \frac{108}{328} \cdot (300 \cdot 10^{-3}) = 98.7 \text{ mA}$

$I_{sc} = I_{sc1} + I_{sc2}$

$\underline{I_{sc} = 1.56 \cdot 10^{-3} + 98.7 \cdot 10^{-3} = 100 \text{ mA}}$

finally. use ohm's law to get the thevenin equivalent

$\Delta V = IR \rightarrow R = \frac{\Delta V}{I} = \frac{32.8}{100 \cdot 10^{-3}} = 328\ \Omega$